11/6/18

Wave Moretto

Lecture #12: Statistical Interference

inferential summary

pop	unknown pop. quantity of main interest	μ = pop. mean equil. temp
imag. data → sample	estimate of μ	$\bar{y} = 25.0°C$
	give or take for $\bar{y}$ as estimate of μ	$\hat{SE}_{IID}(\bar{y}) = 0.27°C$
	95% conf. int. for μ	$(24.5°C, 25.6°C)$

hypothetical IID sample $\left[\int\right]$ $n=25$

imag data

$\begin{bmatrix} 25.0°C \\ 24.7°C \\ \vdots \\ \vdots \end{bmatrix}$ $M \to \infty$ ⟵ mean $\bar{y}$ = ? (ex. 24.7 ...)

- Long run mean: EV of $\bar{y} = \mu$
- Long run est. SD: $\hat{SE}$ of $\bar{y} = \cancel{\sigma}s/\sqrt{n} = \frac{1.34°C}{\sqrt{25}} = 0.27°C$
- Long run hist: $\underset{\mu}{\underline{\quad|\quad}}$ (*) $\hat{SE} \frac{\cancel{\sigma}s}{\sqrt{n}} = 0.27°C$

* formula 3 R-22

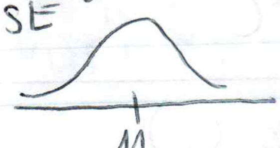
$$\begin{pmatrix}\text{expected} \\ \text{value } \bar{y}\end{pmatrix} = \begin{pmatrix}\text{EV} \\ \text{of } \bar{y}\end{pmatrix} = \boxed{E_{IID}(\bar{y}) = \mu}$$

* formula 4 R-22

$$\begin{pmatrix}\text{estimated} \\ \text{standard} \\ \text{error of } \bar{y}\end{pmatrix} = \hat{SE} \text{ of } \bar{y} = \boxed{\hat{SE}_{IID}(\bar{y}) = \frac{\cancel{\sigma}s}{\sqrt{n}}}$$

• On the basis of this data set, we think that μ is around 25°C ($\bar{y}$), give or take about 0.27°C ($\hat{SE}(\bar{y})$), and a 95% CI for μ runs from 24.5°C to 25.6°C

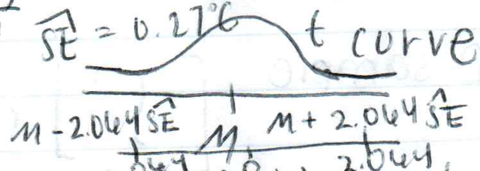
(*) $\hat{SE}$ 0.27°C



(CLT) L-124 • W.S. Gosset (1908)

long run hist of $\bar{y}$

• long run hist. of $\bar{y}$, accounting for uncertainty in σ (n obs)

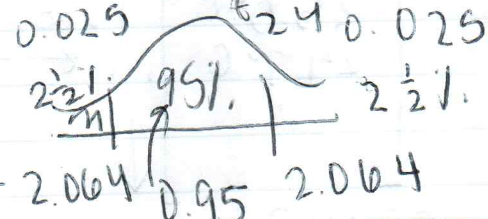


→ longer tails

• t_{n-1} ≈ t curve w/ n degrees of freedom

• L-(142) t table 1

n = 25 t_{24}



0.95 =

• P($\mu - 2.064\hat{SE} < \bar{y} < \mu + 2.064\hat{SE}$)

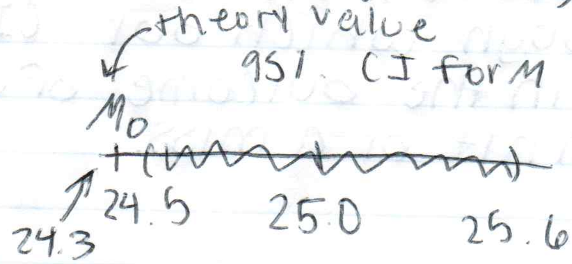
• Neyman's confidence trick

$\mu < \bar{y} + 2.064\hat{SE}$ $\mu > \bar{y} - 2.064\hat{SE}$

↳ $\bar{y} - 2.064\hat{SE} < \mu < \bar{y} + 2.064\hat{SE}$

• P($\bar{y} - 2.064\hat{SE} < \mu < \bar{y} + 2.064\hat{SE}$)
= 0.95 = 95% →
$\bar{y} \pm 2.064\hat{SE}(\bar{y})$ is a 95% confidence interval (CI) for μ

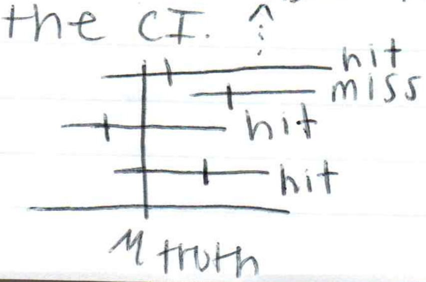
$25.0°C \pm 2.064(0.27°C) = (24.5, 25.6)°C$



- informally we're pretty sure μ is somewhere between 24.5 & 25.6
- Since theory value $\mu_0 = 24.3$ for μ is not in the 95% confidence interval for μ, ① we conclude that the theory is probably wrong
② The difference between 24.3 + 25.0 is statistically significant (stat sig) at the 95% level of confidence

Diff question

- Is the difference between 24.3 + 25.0 practically significant (pract sig)?
- Difference between theory value μ_0 and data value $\bar{y}$ statsig $\Leftrightarrow$ difference is hard to attribute to unlucky random sampling $\Leftrightarrow$ theory probably wrong
- Is confidence the same as probability?
- 95% CI for μ is (24.5, 25.6); does this mean probability $P_F(24.5 < \mu < 25.6) = 95\%$
frequentist $\Rightarrow$ No μ is a fixed unknown number that is either in or out of the CI. ?



~~asp.~~ Neyman guarantees 95% of these intervals will be hits

- Neyman's/our confidence is in the process through which our CI was made, not in the outcome of whether our CI is a hit or a miss.