10/23/18

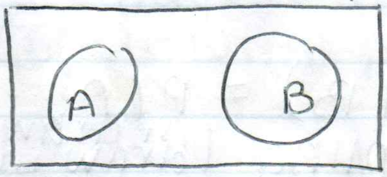
Wave Moretto

AMS Lecture #18: Probability

R-37 – summary of probability rules

- working with or

No-overlap 100%

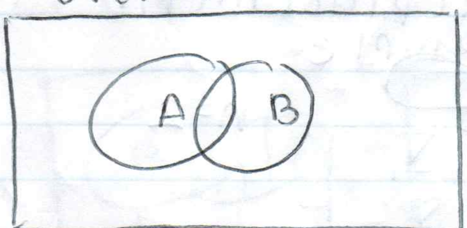


$P(A \text{ or } B) =$

$P(A) + P(B)$

special case of $\boxed{\text{or}}$, when A and B don't overlap = mutually exclusive

overlap



$P(A \text{ or } B) =$

$P(A) + P(B) - P(A \text{ and } B)$

general rule for $\boxed{\text{OR}}$

- working with and

-2 cases to consider

Pop. → sample

$$\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow[\text{random}]{\text{at}} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \quad n = 2$$

$P(Y_1 = 2 \text{ and } Y_2 = 2) = ?$

case 1: at random w/ replacement (independent identically distributed sampling – IID)

IID (2nd draw across, 1st draw down)

	1	2	9
1	(1,1)	(1,2)	(1,9)
2	(2,1)	(2,2)	(2,9)
9	(9,1)	(9,2)	(9,9)

Q: ELM apply to these 9 possibilities? A: Yes!

$P(Y_1 = 2 \text{ and } Y_2 = 2) = \frac{1}{9}$

$P(A \text{ and } B) \stackrel{?}{=} P(A) \, ? \, P(B)$

$P_{IID}(Y_1 = 2) = \frac{1}{3} = \frac{3}{9}$ $\quad P_{IID}(Y_2 = 2) = \frac{1}{3} = \frac{3}{9}$

$P(Y_1 = 2 \text{ and } Y_2 = 2) = \frac{1}{9}$

$P(Y_1 = 2) = \frac{1}{3}$ $P(Y_2 = 2) = \frac{1}{3}$

$\frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$ ✓

• Theory: $P(A \text{ and } B) = P(A) \cdot P(B)$
works for IID sampling (1st draw and 2nd draw independent)

• Case 2: simple random sampling (SRS)
at random w/o replacement

Pop   sample $n=2$

$$\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow{SRS} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$$

$P_{SRS}(Y_1 = 2 \text{ and } Y_2 = 2) = 0$

$\boxed{SRS}$	2nd draw: 1	2	9
1st draw 1	~~(1,1)~~	(1,2)	(1,9)
2	(2,1)	~~(2,2)~~	(2,9)
9	(9,1)	(9,2)	~~(9,9)~~

Q: Does ELM apply to those 6 possibilities?
A: Yes

$P_{SRS}(Y_1 = 2) = \frac{1}{3} = \frac{2}{6}$

$P_{SRS}(Y_2 = 2) = \frac{1}{3} = \frac{2}{6}$

• $P_{SRS}(Y_1 = 2 \text{ and } Y_2 = 2) = 0$

$\underbrace{P_{SRS}(Y_1 = 2)}_{\frac{1}{3}} \cdot \underbrace{P_{SRS}(Y_2 = 2)}_{\frac{1}{3}} = \cancel{\frac{1}{9}}$

- our theory doesn't work for SRS

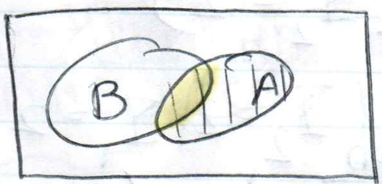
• IID second draw doesn't depend on first draw

• SRS 2nd draw depends on first draw

IID Pascal, Fermot 1650
SRS de Moive (1710)
Bayes (1760)

- Conditional probability: P(B given A)? = P(B|A) ↑ given

$$P(B) = \frac{B}{\square}\ 1$$



P(B given A)

$$= \frac{\cancel{P}\,O\ A \text{ and } B}{A}$$

- Def: $\boxed{P(B|A) = \dfrac{P(B \text{ and } A)}{P(A)}}$

- More formally: $P(B|A) = \left[\dfrac{P(B \text{ and } A)}{P(A)} = \dfrac{P(A \text{ and } B)}{P(A)} \right.$
if $P(A) > 0$
undefined if $P(A) = 0$

- $P(B|A) = \dfrac{P(A \text{ and } B)}{P(A)}$ mult. by P(A)

$P(A \text{ and } B) = P(A)[P(B|A)]$

- $P(A|B) = \dfrac{P(A \text{ and } B)}{P(B)}$

$P(A \text{ and } B) = P(B) \cdot P(A|B)$

general product rule for and

↑(chain rule for probability)

- Special case (IID):
- Def: A, B are independent if information

about A does not change the chances for B and vice versa

if A, B independent

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

IID $\underset{\uparrow}{\ }$ $Y_1 = 2$ $\quad$ $Y_2 = 2$

$= P(Y_1 = 2) \cdot P(Y_2 = 2 | Y_1 = 2)$

- If A and B are independent then $P(B|A) = P(B)$ and $P(A|B) = P(A)$
- $P_{IID}(Y_1 = 2 \text{ and } Y_2 = 2)$

$= P_{IID}(Y_1 = 2) \cdot P_{IID}(Y_2 = 2 | Y_1 = 2)$

$= \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$ ✓

$P_{SRS}(Y_1 = 2 \text{ and } Y_2 = 2)$

$= P_{SRS}(Y_1 = 2) \cdot P_{SRS}(Y_2 = 2 | Y_1 = 2)$

$= \frac{1}{3} \cdot 0 = 0$ ✓

$$\begin{array}{cc} \text{MLP} & \text{G} \\ \end{array}$$
$$\begin{bmatrix} Y & F \\ Y & M \\ N & F \\ \vdots & \vdots \end{bmatrix} \quad n = 106 \xrightarrow{\text{(sort)}} \begin{bmatrix} Y & F \\ \vdots & \vdots \\ Y & F \\ N & F \\ \vdots & \vdots \\ N & F \\ Y & M \\ \vdots & \vdots \\ Y & M \\ N & M \\ N & M \end{bmatrix} \begin{array}{c} 29 \\ \\ \\ 20 \\ \\ \\ 52 \\ \\ \\ 5 \\ \hline 106 \end{array}$$

1 row for each individual

qual, categorical data analysis

G \ MLP	Y	N	
F	29	20	49
M	52	5	57
	81	25	106

2×2 contingency table

Q Are gender and MLP independent in this data set?

A: Imagine choosing a person at random from 106 people (ELM? yes)

$P(yes) = \frac{81}{106} = 76\%$

$P(yes|female) = \frac{29}{49} = 59\%$

$P(yes|male) = \frac{52}{57} = 91\%$

Since $91\% \neq 59\% \neq 76\%$ gender and m/p are strongly dependent in this data set. And, are associated.

R-51 - case study (identical structure on HW2 #4)

- Outcome (Y): Y death penalty, N no dp
- "treatment" (X): W white B black (non white)

$P(DP) = \frac{36}{326} = 11\%$ Pg. L-114

$P(DP|DW) = \frac{19}{160} = 11.9\%$ a bit of surprise

$P(DP|NW) = \frac{17}{166} = 10.2\%$

$P(DP|VW) = \frac{30}{214} = 14\%$

$P(DP|VW, DW) = \frac{19}{151} = 16.5\%$ and

$P(DP|VB) = \frac{6}{112} = 5.4\%$

$P(DP|VB, DW) = \frac{0}{9} = 0\%$

Simpson's paradox