Lecture #14: Sample Size Determination

- On HW can do 1 and 3 by end of clay 2
- HW due 11/25 @ 11:59 PM
- Today L-171

<table>
<thead>
<tr>
<th>Person #</th>
<th>A</th>
<th>B</th>
<th>diff A-B</th>
<th>(Systolic BP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>177</td>
<td>159</td>
<td>-18</td>
<td>n=1000</td>
</tr>
<tr>
<td>2</td>
<td>155</td>
<td>157</td>
<td>+2</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>201</td>
<td>201</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Mean \( \bar{y} = -1 \) mm Hg
SD \( s = 10 \) mm Hg

The diff of 1 mm Hg is not pract sig (clinically, medically)

95\% CI

\[ \bar{y} \pm 1.96 \frac{s}{\sqrt{n}} \]

\[ -1 \pm 1.96 \frac{10}{\sqrt{1000}} = -1 \pm 0.32 \]

So diff is not stat sig

- Why is stats sig not pract sig?
  - Too much data (made SE too small)

- \( D = A - B \)
  - [ ] n=10
  - Is this diff pract sig?
    - Yes

Mean \( \bar{y} = -10 \) mm Hg
SD \( s = 20 \) mm Hg

\[ \bar{y} \pm (t_{0.05}) \frac{s}{\sqrt{n}} \]

\[ -10 \pm 2.262 \left( \frac{20}{\sqrt{10}} \right) \]

\[ -24 - 10 + 4 \]

Not stats sig

- To little data (made SE too big)
- Solution choose \( n \) so that stats sig = pract sig
R-58, Discussion Section 5 #2

M = pop mean CA concentration

null H₀: \( M = 32 = M₀ \) (theory 1)

alt \( M = 31.5 = M_A \)

H₁: (theory 2)

Q: How big should \( M \) be to reliably discriminate between theories 1 and 2?

Sample size determination

1. Confidence Interval Approach

   Take \( n \) obs and build 95\% CI

   \[
   \bar{y} \pm t_{0.95(2)} \frac{s}{\sqrt{n}} \]

   place along \( t_{n-1} \)

   curve w/ L-174

   0.95 in middle, 0.05 in 2 tails combined

   95\%. CI = 100 \((1-\alpha)\) / \( \alpha = 0.05 \)

   Suppose past experience makes you think

   \( \bar{y} \) will come out around 1.8

   \( M₀ = 31.5 \)

   \( M₀ = 32 \) - Falls just outside CI

   \[
   \bar{y} - t_{0.95(2)} \frac{s}{\sqrt{n}} \quad \bar{y} + t_{0.95(2)} \frac{s}{\sqrt{n}}
   \]

   \[
   M₀ = M_A \left[ \frac{t_{0.95(2)}}{n-1} \right] \frac{s}{\sqrt{n}}, \text{ solve for } n
   \]

   \( (32) = (31.5) \)

   get

   \[
   n = \left[ \frac{t_{0.95(2)}}{n-1} \right] \frac{s^2}{(M₀ - M_A)^2}
   \]

   \( s \uparrow \) (noise level data \( \uparrow \)) \( n \uparrow \)

   \( |M₀ - M_A| \uparrow \) (theories easier to tell apart) \( n \uparrow \)

   \( \alpha \downarrow \) (confidence level of interval \( \uparrow \)) \( n \uparrow \)
needs to be solved iteratively; start on right hand side (rhs) \( n_0 = 32 \), solve for \( n \), look up new \( t \) with this \( n \) and put it in rhs, solve again, repeat as needed (usually 2 steps required).

\[ s = 1.8 \text{ mmol/kg} \]
\[ |M_0 - M_A| = 132 - 31.51 = 100.5 \text{ mmol/kg} \]
\[ t = \frac{100.5}{1.8} 
\]

\[ n = \frac{(1.96)}{(0.5)}^2 \approx 49.8 = 50 \]

\[ \text{(always round up)} \]

\[ n = \frac{(2.010)}{(0.5)}^2 \approx 62.4 = 63 \]

\[ n = \frac{(2.007)}{(0.5)}^2 \approx 63 \]

\[ n = 53 \]

Significance/hypothesis testing approach

\[ H_0: M = M_0 \text{ (theory)} \]
\[ H_A: \{ M \neq M_0 \text{ (2 sided alt)} \text{ (2 tailed test)} \\
\] \[ M > M_0 \text{ (1 sided alt)} \text{ (1 tailed test)} \]

* Type I error = false rejection of null
* Type II error = false acceptance of null
Neyman-Pearson: you get a random sample of \( n \) obs and cover again, sometimes by chance when \( H_0 \) true you would (w/o meaning to) fall into type I error, if \( H_0 \) false type 2 error w/o meaning, both error probabilities small.

\[ P(\text{reject } H_1 | H_0 \text{ true}) = P(\text{type I error}) \]
\[ = \alpha = \text{sig. level of test} \]

Want small.

Ex): if \( P(\leq 0.01) \rightarrow \text{reject } H_0 \)

\[ P(\text{don't reject } H_0 | H_0 \text{ false}) = P(\text{type 2 error}) = \beta \text{ (beta)} \]

Want small.

So want \( (1 - \beta) \) to be big: \( -\beta \)

Power of test.

To force \( t \) test to have \( P(\text{type I error}) \) to move \( \alpha \) and \( P(\text{type II error}) \) no more than \( \beta \) this is how many obs. you need at least

For 1-tailed test: \( 2 \) for 2-tailed test:

\[ n = \left[ \frac{t(1-\alpha/2)(\beta)}{s} + t(1-\beta/2)(1)} \right]^2 \frac{s^2}{(\mu_0 - \mu_A)^2} \]

How to choose \( \beta \) hard work.

\[ \alpha = 0.05 \text{, } 0.01 \text{, } 0.2 \]

Ex): calcium \( s = 1.8 \text{ null } = \mu_0 = 32 \)

\[ \text{calc+ } \mu_A = 31.5 \text{ null - anat } = 0.5 \]

\[ \alpha = 0.05 \text{ power } = 1 - \beta = 0.9 \text{ so } \beta = 0.1 \]
1. Start with a hypothesis test for two samples.

\[ \alpha \approx 0.9 \]

\[ n = \frac{[1.96 + 1.28]^2 (1.8)^2}{(0.5)^2} = 136 \]

2. Another hypothesis test for two samples.

\[ t_{135} \approx 1.35 \]

\[ -1.98 \quad 1.98 \]

\[ n = \frac{[1.98 + 1.29]^2 (1.8)^2}{(0.5)^2} = 139 \quad \text{stop} \]

3. \( n = 139 \) so stop.

- For two always enough for just 2 steps.

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- The 2 sample problem.
- 2 cases to continue:
  1. Paired comparisons
  2. Analysis of 2 independent samples

- Paired comparisons:
  - Example: RCP+ cortex size case study
  - Created matched pairs

- But attention in inference (stat. sig) focuses on 1-sample column of differences. This converts 2 sample to 1-sample.

- Repeated measures - measure same variable on n individuals at 2 cliff points in time (longitudinal)
mean $\overline{a}$

Standard deviation $\sigma$

follows normal curve

Inferential Summary

<table>
<thead>
<tr>
<th>Unknown pop. q.of main interest</th>
<th>$M_d = \text{pop. mean diff (cm)}$ in $\overline{a}$ vs $\overline{b}$ length</th>
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</table>

**Estimate**

- $\overline{a} = 3.30 \text{ cm}$

**Give or take for $M_d$**

- $SE(\overline{a}) = 0.97 \text{ cm}$

**95\% CI for $M_d$**

- $\overline{a} \pm 2.262 \times SE(\overline{a}) = (1.11, 5.49)$

1. EV of $\overline{a} = E[IIFO(\overline{a})] = E[IIFO(\overline{y})] = M_d$
2. $SE$ of $\overline{a} = \sqrt{\frac{SE(IIFO \overline{a})}{n}} = \frac{sd}{\sqrt{n}}$
3. $\overline{a} \pm (t_{n-1}) \frac{s}{\sqrt{n}}$
4. $3.30 \pm (2.262)(0.97)$
5. $3.30 \pm 2.202$

- $0$ not in $95\%$ CI, so diff is statsig
- Can now go entire HW 3 (this is like problem 4)