11/13/18
Wave
Moretto

Lecture #14: Sample Size Determination

- on HW can do 1 and 3, by end of day 2
- HW due 11/25 @ 11:59 PM
- Today L-17 1

person #	B	A	diff. A − B
1	177	159	−18
2	155	157	+2
⋮			
n	201	201	0

(Systolic BP)
$n = 1000$

mean $\bar{y} = -1$ mmHg
SD $s = 10$ mmHg

The diff. of 1mm Hg is not pract sig (clinically, medically)

95% CI

$\bar{y} \pm t^{0.95}_{999} \frac{s}{\sqrt{n}}$ (here $t = z$)

$-1 \pm 1.96 \frac{10}{\sqrt{1000}} = 0.32$ ~~too big~~!

95% CI: (−1.6, −1, −0.4) 0

so diff is stat sig

*Why is statsig not practsig?
↳ Too much data (made SE too small)

$D = A - B$

[ ] $n = 10$

mean $\bar{y} = -10$ mmHg
SD $s = 20$ mmHg

- Is this diff. practsig? − Yes
- Statsig? → No

$\bar{y} \pm (t^{0.95}_{9}) \frac{s}{\sqrt{n}}$

$(-10) \pm 2.262 \left(\frac{20}{\sqrt{10}}\right)$

$= -10 \pm 14$ mmHg

(−24, −10, +4), 0

↳ Not statsig

- To little data (made SE too big)
- Solution: choose (n) so that statsig $\doteq$ practsig

R-58, Discussion Section 5 #2

μ = pop mean ca concentration

null H_0: $\mu = 32 = \mu_0$ (theory 1)

alt $\mu = 31.5 = \mu_A$

H_A: (theory 2)

Q: How big should n be to reliably discriminate between theories 1 and 2?

- sample size determination

① Confidence Interval Approach

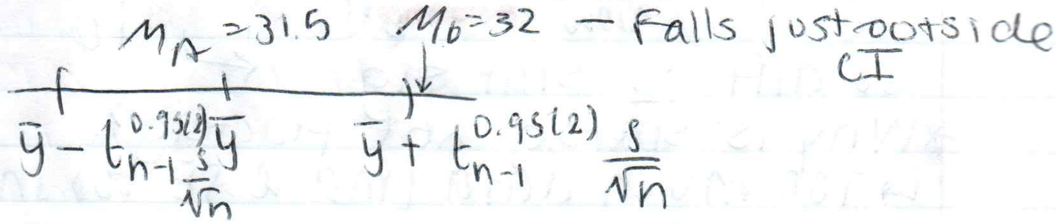
take ⓝ obs and build 95% CI

$\bar{y} \pm t_{n-1}^{0.95(2)} \frac{s}{\sqrt{n}}$ → place along t_{n-1} curve w/ L-174

0.95 in middle, 0.05 in 2 tails combined

95% CI = 100 $(1-\alpha)$% (α = 0.05)

suppose past experience makes you think ⓢ will come out around 1.8



$\mu_0 = \mu_A + \left[t_{n-1}^{0.95(2)} \right] \frac{s}{\sqrt{n}}$, solve for n

(32) = (31.5)

get

$$n = \frac{\left[t_{n-1}^{(1-\alpha)(2)} \right]^2 s^2}{(\mu_0 - \mu_A)^2}$$ ⊛

- $s \uparrow$ (noise level data $\uparrow$) $n \uparrow$
- $|\mu_0 - \mu_A| \uparrow$ (theories easier to tell apart) $n \downarrow$
- $\alpha \downarrow$ (confidence level of interval $\uparrow$) $n \uparrow$

(*) needs to be solved iteratively: start on right hand side (rhs) w/ $n = \infty$ ($t = z$ (normal curve)), solve for n, look up new t w/ this n and put it in rhs, solve again, repeat as needed (usually 2 steps required).

ex) $s = 1.8$ mmol/kg

$|\mu_0 - \mu_A| = |32 - 31.5| = 0.5$ mm/kg

$\alpha =$ ~~0.025~~ 0.5 + 95% CI

① $n = \frac{(1.96)^2 (1.8)^2}{(0.5)^2} = 49.8 = 50$ (always round up w/ n)

② try $n = 50$

$t_{n-1}^{0.95(2)} = 2.010 \quad n = \frac{(2.010)^2 (1.8)^2}{(0.5)^2} = 52.4 \rightarrow 53$

③ try $n = 53$

$t_{n-1}^{0.95(2)} = 2.007 \quad n = \frac{(2.007)^2 (1.8)^2}{(0.5)^2}$

$=$ (53) ⇒ done

② significance/hypothesis testing approach

H_0 (null): $\mu = \mu_0$ (theory 1) $\mu_0 = 32$

H_A (alt):
- $\mu \neq \mu_0$ (2 side alt) (2 tailed test)
- [$\mu > \mu_0$, $\mu < \mu_0$] (1 side alt) (1 tailed test)

truth

data says	H_0 false	H_0 true
reject H_0	good	type I (α) error
don't reject H_0	type II (β) error	good

- Type I error = false rejection of null
- Type 2 error = false acceptance of null

- Neyman + Pearson: you get a random sample of ⓝ obs and cover again, sometimes by chance when H_0 true you would (w/o meaning to) fall into type I error, if H_0 false type 2 error w/o meaning, both error probabilities small

- P(reject H_0 | H_0 true) = P(type I error) = α = sig. level of t_{eH}
  ↑ want small

ex) if $P(\leq 5\% \rightarrow$ reject $H_0)$

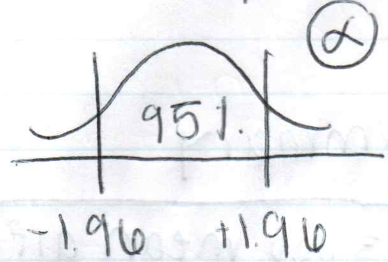
- P(don't reject H_0 | H_0 false) = P(type 2 error) = β (beta)
  ↑ want small
- So want $(1-\beta)$ to be big : $(-\beta)$ = **power** of test
- To force t test to have P(type I error) to move ⓐ and P(type II error) no more than ⓑ this is how many obs. you need at least
① For 1-tailed test 2 for 2 tailed t_{TSH}

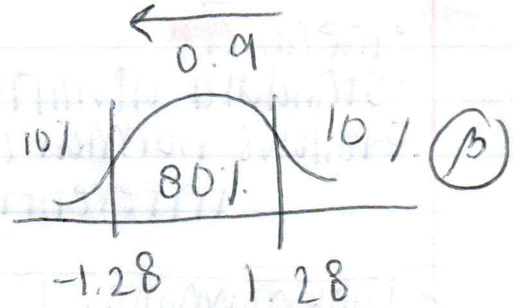
$$n = \frac{\left[t_{n-1}^{(1-\alpha)(2)} + t_{n-1}^{(1-\beta)(1)} \right]^2 s^2}{(\mu_0 - \mu_A)^2}$$

- How to choose β → hard work
  $\alpha = 0.05, 0.01$ $\beta = 0.1, 0.2$

ex) calcium $s = 1.8$ null $= \mu_0 = 32$
alt $\mu_A = 31.5$ $|\mu_0 - \mu_A| = 0.5$
$\alpha = 0.05$ power $= 1-\beta = 0.9$ so $\beta = 0.1$

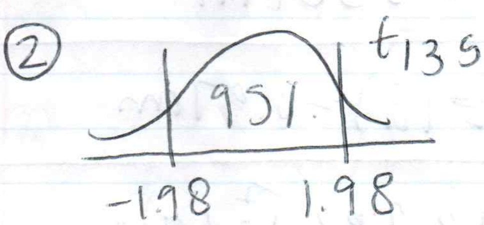
① start w/ z

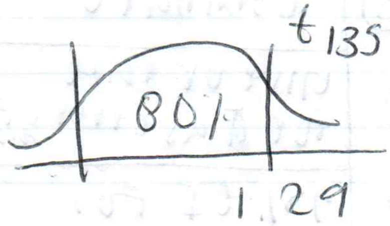




$$n = \frac{[1.96 + 1.28]^2 (1.8)^2}{(0.5)^2} = 136$$

②





$$n = \frac{[1.98 + 1.29]^2 (1.8)^2}{(0.5)^2} = \underline{139} \text{ stop}$$

③ n = 139 so stop
- for HW always enough for just 2 steps

- The 2 sample problem
- 2 cases to continue

① Paired comparisons
② Analysis of 2 independent samples

- Paired comparisons

ex) Rat cortex size case study
- created matched pairs
- But attention in inference (stat. sig) focuses on 1-sample column of differences
this converts 2 sample to to 1-sample
- Repeated measures - measure same variable on n individuals at 2 diff. points in time (longitudinal)

- mean $\bar{d}$
- standard deviation s_d
- follows normal curve

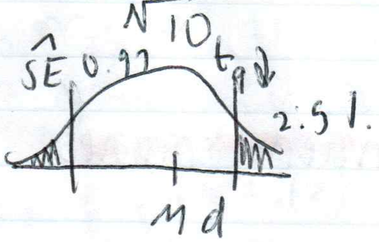
inferential summary

(P)	unknown pop. q of main interest	μ_d = pop. mean diff (cm) in (H) vs. (F) length
(S)	estimate	$\bar{d} = 3.30$ cm
↑	Give or take for $\bar{d}$ as error of μ_d	$\hat{SE}(\bar{d}) = 0.97$ cm
(ID) ↓	95% CI for μ_d	$\bar{d} \pm 2.262\ \hat{SE}(\bar{d}) =$ (1.11, 5.49)

EV of $\bar{d} = E_{IID}(\bar{d}) = E_{IID}(\bar{y}) = \mu_d$
$= \mu_d$

$\hat{SE}$ of $\bar{d} = \hat{SE}_{IID}(\bar{d}) = \hat{SE}_{IID}(y) = \frac{s_d}{\sqrt{n}}$

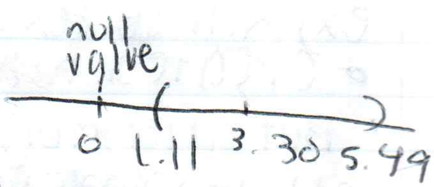
$= \frac{3.06}{\sqrt{10}} = 0.97$ cm



$\bar{d} \pm \left(t_{n-1}^{95\%}(I)\right)\frac{s}{\sqrt{n}}$

$3.30 \pm (2.262)(0.97)$

3.30 ± 2.19



- 0 not in 95% so diff is stat sig
- Can now do entire HW 3 (this is like problem 4)