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November 8th
Thursday

$\boxed{\text{CIs for BINARY DATA SETS}}$

12 animals - LAB RATS (Ran through maze to food)
If they couldn't smell food, the rats will be split (50-50).

But 83% turned towards food.

Practically sig. difference

How surprising is ~~get 80~~ 83% if 50% was expected?
does 83% lie in the give and take of 50%?

Pop

$\begin{bmatrix} 1s \\ \vdots \\ 0s \\ \vdots \end{bmatrix}$ $N = $ ~~∞~~ ? (big)

like SRS

SRS ≈ IID

$N >> n$

left?

$\begin{bmatrix} 1s \\ \vdots \\ 0s \end{bmatrix}$ $n=12$

$\bar{y} = \hat{p} = \frac{10}{12} = 83\%$

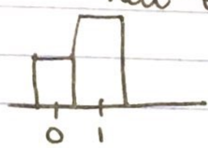
hat = estimate

imag data

$\begin{bmatrix} 83\% \\ 75\% \\ \vdots \\ \vdots \end{bmatrix}$ imagine getting many $\hat{p}$s

$\mu(p) = ?$

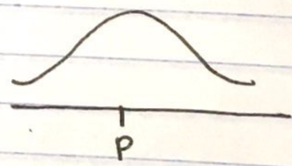
σ : ?



Long Run Mean (EV of $\hat{p}$)
$= p$ (ex of $\bar{y}$ IID)
Long Run SD

p - proportion
(μ in disguise)

SI model



INFERENTIAL SUMMARY

pop.	unknown quantity of interest	p = population % of lab animals that smell food (left)
sample	estimate	$\hat{p} = 83\%$
	give and take for $\hat{p}$ estimate of p	~~$\hat{p}$~~ $SE(\hat{p}) = 11\%$
	95% CI for p can estimate	$\hat{p} \pm 1.96\sqrt{\frac{(1-\hat{p})\hat{p}}{n}}$

① EV of $\hat{p} = E_{IID}(\hat{p}) = E_{IID}(\bar{y}) = \mu = p$

$\boxed{\therefore E_{IID}(\hat{p}) = p}$

② SE of $\hat{p} = SE_{IID}(\hat{p}) = SE_{IID}(\bar{y}) = \frac{\sigma}{\sqrt{n}}$ ← we don't know ~~th~~ what σ is

If pop is only 2 values in it, pop SD $= \sigma =$

$$(\text{larger value}) - (\text{smaller value}) \times \sqrt{\text{proportion large val} \cdot \text{proportion of small val}}$$

proportion of 1s $= \mu = p$ &
proportion of 0s $= 1 - p$

1 - large value
0 - small value

pop. SD $= \sigma = (1-0)\sqrt{p \cdot (1-p)}$

with 0s and 1s $\sigma = \sqrt{p(1-p)}$

But, we know n, not p
if $\hat{p}$ ~~≈ p~~ is a good guess for p we should be able to replace ps with $\hat{p}$

☆ so $SE_{(\hat{p})} = \frac{\sqrt{p(1-p)}}{\sqrt{n}} = SE_{IID}(\hat{p})$

BINARY OUTCOMES

↓

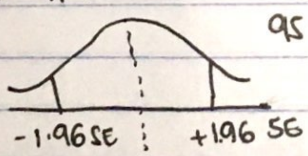
$SE_{IID}(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.83)(0.17)}{12}}$

$= 11\%$

Devil's advocate position:
·83% attributed to unlucky random sampling.

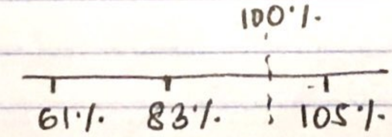
③ CLT doesn't apply in this ~~theory~~ problem

③ SE = 11%



95% of the time, $\hat{p}$ will differ from p by no more than 1.96 SE

83% ± 1.96 (11%)



Chop it off @ 100%.

approximate 95% interval (when n is large)

- ±1.96 SEs from mean($\hat{p}$)

for small $\underline{\underline{n}}$ the values can go above/below 100% / ~~1~~0%

@ 95% CI, 50% (devil's adv. # outside range) ~~outside~~ theory $\underline{\underline{\text{not true}}}$.

Hypothesis Tests (Neyman and Pearson)
Significance Tests: (Fischer)

→ H_0 : Null Hypothesis → μ_0 - Null Hypothesis value
↳ absence

① The difference b/w μ_0 and μ $\underline{\underline{\text{is}}}$ due to unlucky random sampling.

H_A : Alternative Hypothesis: → $\mu \neq \mu_0$

② Difference between μ and μ_0 is real.

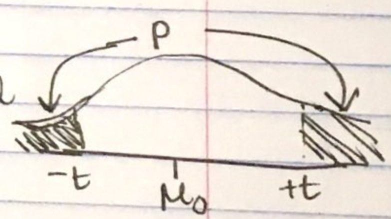
Temporarily pretend H_0 is true,

How data came out v/s How data should have come out if null true

accept alt.

accept null

Long Run hist (if null were true)



How measure discrepancy

$$\frac{\bar{Y} - \mu_0}{SE \text{ of } \bar{Y}} = \frac{\text{"signal"}}{\text{"noise"}} = \frac{25.0°C - 24.3°C}{0.27°C} = +2.59 = t$$

SE of $\bar{Y}$ (if null is try) → $\frac{s}{\sqrt{n}}$

↑ t tests

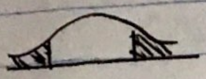
is 2.6 big enough ~~to be~~ for null to be wrong?

P value - numerical measure of surprise (if null were true)

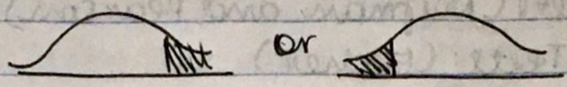
t - way in the tail, → p value small

↳ if this is the case, throw away null

2 tailed p values



1 tailed p value



if $p \leq 5\%$ result is statistically sig. → Reject null

Hypothesis testing with 2 sided alt = same as CI (95%)

2 tailed test = 2(1 tailed test)