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TAKE HOME MIDTERM

☆ P(1 or more T-S for a family of 5)

Both parents are carriers.

$= 1 - P($ ~~not~~ 0 TS Babies $)$

$= 1 - P(1^{st}\text{ not TS and } 2^{nd}\text{ not TS} \ldots\ldots 5^{th}\text{ not TS})$

TO MULTIPLY, BOTH EVENTS NEED TO BE INDEPENDENT

11D

$1 - \left(\left(1-\frac{1}{4}\right) \times \left(1-\frac{1}{4}\right) \cdots \left(1-\frac{1}{4}\right)\right)$

$= 1 - \left(1-\frac{1}{4}\right)^5 = 0.76 = 76\%$

# of T-S babies

0
1
2 one or
3 more
4
5

If ELM applies here

$p(1 \text{ or more}) = \frac{5}{6}$

But ELM doesn't apply. (Venn diagram L-104)

$P(\text{Exactly 1 T-S}) \neq P(\text{Exactly 2 T-S}) \neq P(\text{Exactly 3 T-S})$

PROBABILITY MODELS FOR SUMS

Roulette

$P(\text{coming out ahead, 1 \$1 bet on (A)}) = \frac{1}{38} = 2.6\%$

single #

$P(\text{coming out ahead, 1 \$1 bet on (B)}) = \frac{2}{38} = 5.2\%$

split

PROBABILITY MODEL FOR ROULETTE:

observed spins

wheel pair → $\begin{bmatrix} -1 \\ -1 \\ -1 \\ +35 \\ \vdots \end{bmatrix}$ $n=1000$ → sum \$ = ?

11D

variable:
your net gain

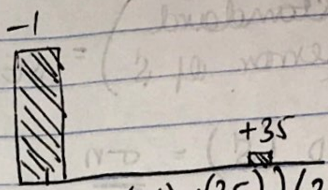
56+53-29

population

all possible spins

Your net gain

$$N=38 \quad \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ +35 \\ \vdots \\ \vdots \end{bmatrix} \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ \boxed{6} \\ \vdots \\ 38 \end{array}$$



pop. histogram

(μ) pop mean $((-1)+(-1)\cdots(-1)+(35))/38 = -2/38 = 1/19 \approx 5\%$

(σ) pop SD:

Meaning of mean: "with every \$1 bet on a single #, I expect to gain ($-\$0.05$), give or take $\sigma = \$5.76$

$$\sigma = \sqrt{\frac{(1-(-0.05))^2 \cdots + (35-0.05^2)}{38}}$$

σ = pop SD = (larger value − smaller value) $\sqrt{(\text{proportion of larger values})(\text{proportion of smaller val})}$

$$\Rightarrow \sigma = \underbrace{(+35-(-1))}_{36}\sqrt{\frac{1}{38}\cdot\frac{37}{38}} = \$5.76$$

(Your net gain ~~off~~ after 1000 \$1 bets on a single #)
real world

↓ is exactly like (analogy)

ESSENCE OF APPLIED MATH

(the sum S of $n=1000$ IID draws from this population)
math and computing - probability models

Range of possible values for S

-1000 ———— +35000

$+35\left(\frac{1}{38}\right)$

$-1\left(\frac{37}{38}\right)$

$\frac{1000}{38} = 26$

$26(+35) + 974(-1) = -64$

$28(+35) + 972(-1) = \$+8$

$29(+35 + 971(-1))$

long run

$$\begin{bmatrix} \$-64 \\ \$+8 \\ \vdots \\ \vdots \end{bmatrix} M$$

$\begin{pmatrix}\text{long run}\\ \text{mean of } S\end{pmatrix} = \begin{pmatrix}\text{Expected}\\ \text{value of } S\end{pmatrix} = \text{EV of } S = E_{IID}(S) = n \times \mu$ = # of draws × population mean.

$\begin{pmatrix}\text{long run}\\ \text{SD of } S\end{pmatrix} = \begin{pmatrix}\text{Standard}\\ \text{error of } S\end{pmatrix} = \text{SE of } S$ ← give or take for S

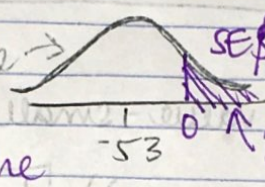
$= SE\ IID\ (S) = \sigma n$

N X
μ X
σ as $\sigma\uparrow$ $SE(S)\uparrow$
n as $n\uparrow$ $SE(S)\uparrow$

after 1000 \$1 bets on single #, I expect to gain (\$-53) (EV) give or take

Central limit theorem:

① As long as n is large draws (IID) will look like



② The ~~smaller n its~~ closer the pop. hist. is to the standard, the smaller n needs to be

③ If pop. hist is normal to begin with, the long run hist. of S in imaginary data set is normal for $n \geq 1$

SPLIT