

Tuesday 13 November 2018

How to decide how much data is enough?

statsig \neq Pract. sig
ex New drug for Hypertension

Person	Before	After
1		
2		
⋮		
n		

$$\begin{aligned} \text{mean } (\bar{y}) &= -1 \text{ mmHg} \\ \text{SD } (s) &= 10 \text{ mmHg} \end{aligned} \left. \vphantom{\begin{aligned} \text{mean } (\bar{y}) &= -1 \text{ mmHg} \\ \text{SD } (s) &= 10 \text{ mmHg} \end{aligned}} \right\} \text{ OF DIFF (A-B)}$$

Practically significant? NO.

Stat sig?

Devil's advocate: Drug no good ($\bar{y} = 0$)

↳ results due to unlucky rand. sampling

$$\bar{y} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} \quad (\text{here } t = 2)$$

$$-1 \pm 1.96 \frac{10}{\sqrt{1000}} \quad \text{SE} = 0.32$$

→ 95% CI $\left. \vphantom{\rightarrow} \right\} \begin{array}{c} \text{95\% CI} \\ -1.6 \quad -1 \quad -0.4 \end{array} \left. \vphantom{\rightarrow} \right\} \begin{array}{l} \text{(Devil's advocate's} \\ \text{position) not on} \\ \text{95\% CI} \therefore \text{ difference} \\ \text{is stat sig.} \end{array}$

• maybe CI is narrow \therefore doesn't include 0

Q3 Why stat. sig isn't pract. sig? SE tiny because of big sample (1000)

$$D = A - B$$

$\left[\right] n = 10$ Is pract. sig

$$\begin{aligned} \text{mean} &= -10 \text{ mmHg} \\ s &= 20 \text{ mmHg} \end{aligned}$$

$$\bar{y} \pm \left(t_{\alpha/2} \right) \frac{s}{\sqrt{n}}$$

$$-10 \pm 2.262 \left(\frac{20}{\sqrt{10}} \right) = -10 \pm 14 \text{ mmHg}$$

WHY
 Pract sig but not stat sig
 *not enough data

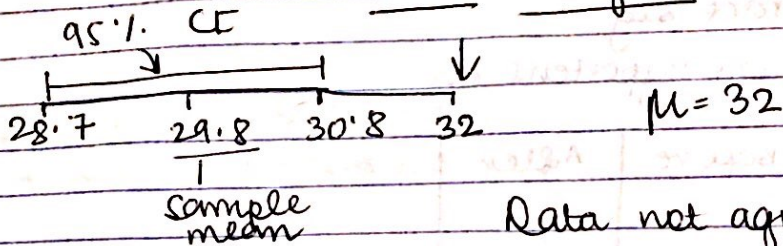
$$\begin{array}{ccc} & \downarrow & \\ -24 & -10 & +4 \end{array}$$

D in 95% CI
 not stat sig.

Zar Case Study SAMPLE SIZE DETERMINATION

Sea Water Calcium conc. 32 mmole/Kg of H₂O

n=13 - why B?



Why 13?

μ = pop mean

Null Hyp = $\mu = 32 = \mu_0$ (Theory 1)

alt H_A : $\mu = 31.5 = \mu_A$ (theory 2)

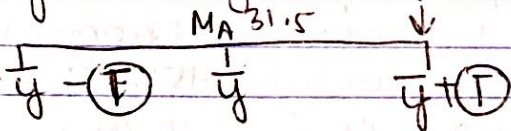
How big should n be for reliability in discrimination between these 2 theories?

Plan: take (n) obs and build 95% CI $(\bar{y} \pm t_{n-1}^{0.95} \frac{s}{\sqrt{n}})$

95% CI = 100(1- α)% ($\alpha = 0.05$)

From past experience \Rightarrow SD(s) = 1.8

If theory 2 is right:



$$n = \left(t_{n-1}^{(1-\alpha)(2)} \right)^2 \frac{s^2}{(\mu_0 - \mu_A)^2}$$

noise \uparrow we need more data (n \uparrow)

$\mu_0 - \mu_A \uparrow$ n \downarrow

s = 1.8

$$|\mu_0 - \mu_A| = |32 - 31.5| = 0.5 \quad \textcircled{1} \quad n = \frac{(1.96)^2 (1.8)^2}{(0.5)^2} = 49.8 = 50$$

$\textcircled{2}$ try n=50

$$t_{49}^{0.95(2)} = 2.010 \quad n = 53$$

$\textcircled{3}$ try n=53 $t_{52}^{0.95(2)} = 2.007$
n = 53

② significance / hypothesis testing

$H_0: \mu = \mu_0$ (theory) $\mu_0 = 32$
null

H_A alt $\left\{ \begin{array}{l} \mu \neq \mu_0 \text{ (2 sided alt) (2 tailed test)} \\ \mu > \mu_0 \\ \mu < \mu_0 \end{array} \right\}$ (1 sided alt) (1 tailed test)

(α) Type 1 error: False rejection of null (when null is true)

(β) Type 2 error: False acceptance of null (when null is false)

$P(\text{reject } H_0 | H_0 \text{ true}) = P \text{ type I error} = \alpha$

$P(\text{don't reject } H_0 | H_0 \text{ false}) = P(\text{type II error}) = \beta$ (Beta)
 \uparrow (want this small)

if β is small, $1 - \beta$ is huge

\uparrow (want this small)

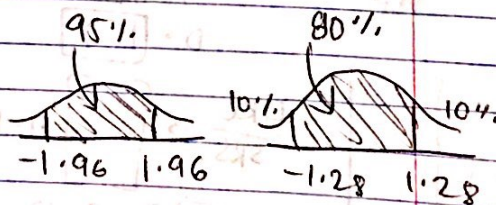
\rightarrow POWER OF THE TEST

$$n = \frac{(t_{n-1}^{(1-\alpha)(2)} + t_{n-1}^{(1-\beta)(2)})^2 s^2}{(\mu_0 - \mu_A)^2}$$

- (1) for 1 tailed test
- (2) for 2 tailed test

ex) calcium $s = 1.8$ | $|\mu_0 - \mu_A| = 0.5$ ① Start w/ 2
2 tailed test $\alpha = 0.05$

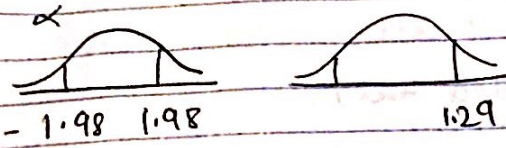
$(1 - \beta) = 0.9$ so $\beta = 0.1$



t for β is always 1 tailed

$$n = \frac{(1.96 + 1.28)^2 (1.8)^2}{(0.5)^2} = 136$$

② $t_{135} = ?$



$$n = \frac{(1.98 + 1.29)^2 (1.8^2)}{0.5^2} = 139$$

③ $n = 139$ so stop

2. Sample inferential problems

How do you compare 2 samples

- ① Paired comparisons
- ② Any Analysis of 2 independent samples

2 sample problem \rightarrow 1 sample problem
(paired comparison) \downarrow

① (T) v/s (C) TAKE DIFFERENCE

② Repeated Measures (2 readings @ different times)
example: Before v/s After on same person

$D = \text{After} - \text{Before} \rightarrow$ to judge stat sig
After and Before \rightarrow for pract sig

③ POP. SAMPLE. ID

<p>$N = ?$</p> <div style="border: 1px solid black; width: 50px; height: 50px; margin: 10px auto;"></div> <p>$M_d = ?$</p>	<p>LIKE SRS \rightarrow</p>	<p>$D = \boxed{H - F}$</p> <div style="border: 1px solid black; width: 50px; height: 50px; margin: 10px auto;"></div> <p>$n = 10$</p> <p>(\bar{d}) $\bar{y} = 3.30$</p> <p>$S = 3.06$</p>	<p>$\begin{bmatrix} 3.30 \\ 3.21 \end{bmatrix}$</p> <p>unknown ϕ of interest</p> <p>ESTIMATE</p> <p>Give or take for \bar{d} at estimate</p> <p>95% CI for M_d</p>	<p>$M_d = \text{pop mean diff } (H - F)$</p> <p>$\bar{d} = 3.30 \text{ cm}$</p> <p>$\hat{\sigma}_{\bar{d}} = 0.97 \text{ cm}$ $= \frac{S}{\sqrt{n}}$</p> <p>$\bar{d} \pm 2.62$ $\frac{2.762}{2.762}$</p> <p>$SE(d) = (1.11, 5.49)$</p>
<p>SE of $\bar{d} = SE_{110}(\bar{d}) = SE_{110}(\bar{y})$</p> <p>$= \frac{S_d}{\sqrt{n}} = 0.97 \text{ cm}$</p>		<p>SE of $\bar{d} = 0.97$</p>		<p>\rightarrow 95% CI = 1.11 — 5.49</p> <p>so diff is stat sig.</p>