this prob. models time: for sums

next time: and means

read: DD (B) ch. 10, 11; LN pp. L-127 → 156

AMS 7 25 Oct 18

today: LN pp. L-119

hwk 2 due by 11.59 pm on Sun 28 Oct 18

$$P(1 \text{ or more T-S babies in family of 5, both parents carriers})$$

$$= 1 - P(0 \text{ T-S babies})$$

$$= 1 - P\left(\begin{matrix}\text{not}\\ \text{T-S}\\ \text{on 1st}\end{matrix} \text{ and } \begin{matrix}\text{not}\\ \text{T-S}\\ \text{2nd}\end{matrix} \text{ and} \cdots \text{and } \begin{matrix}\text{not}\\ \text{T-S}\\ \text{on 5th}\end{matrix}\right)$$

I D

$$= 1 - P\left(\begin{matrix}\text{not}\\ \text{T-S}\\ \text{on 1st}\end{matrix}\right) \cdot P\left(\begin{matrix}\text{not}\\ \text{T-S}\\ \text{on 2nd}\end{matrix}\right) \cdots P\left(\begin{matrix}\text{not}\\ \text{T-S}\\ \text{on 5th}\end{matrix}\right)$$

I ID

$$= 1 - \left(1 - \frac{1}{4}\right) \cdot \left(1 - \frac{1}{4}\right) \cdots \left(1 - \frac{1}{4}\right)$$

$$= 1 - \left(1 - \frac{1}{4}\right)^5 \doteq 0.76 = 76\%$$

# T-s bodies	
0	
1	1 or more
2	
3	
4	
5	

if ELM applies here

$P(1 \text{ or more}) = \frac{5}{6}$

but ELM doesn't apply: see Venn diagram p. L-104

prob. models for sums

R-52

L-119

$\boxed{\text{roulette}}$ $P\left(\text{coming out ahead, 1 \$1 bet on } A\right) = \frac{1}{38}$

ELM? yes $\doteq 2.6\%$

$P\left(\text{coming out ahead, 1 \$ bet on } B\right) = \frac{2}{38}$

$\doteq 5.2\%$

3

population — all possible spins

strategy A $\boxed{6}$

sample — the observed spins

imaginary dataset = repeated sampling dataset

(*) your net gain

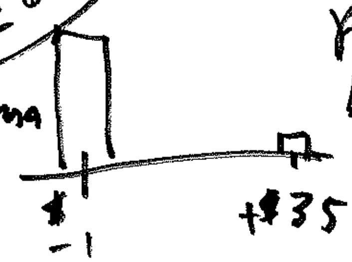
your net gain	
−\$1	1
−\$1	2
⋮	⋮
−\$1	5
+\$35	$\boxed{6}$
−\$1	7
⋮	⋮
−\$1	36
−\$1	0
	00

N = 38 "new"

pop. mean $\mu = -\$0.05$

pop SD $\sigma = \$5.76$ sigma

pop. hist



wheel fair → IID

your net gain

−\$1
−\$1
⋮
+\$35
−\$1
⋮

n = 1,000

sum S = ? ex. −\$64

hyp. IID

n = 1,000

sum S = ? ex. +\$8

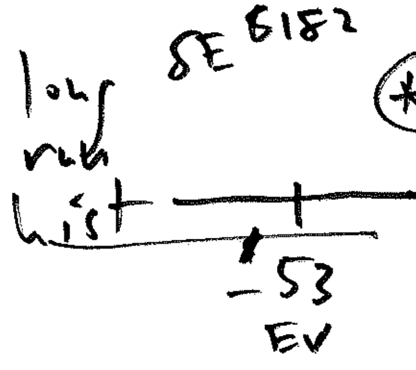
−\$64
+8
⋮

$M \to \infty$

($M \to \infty$) long run mean — EV of S ≐ −\$53

long run SD — SE of S ≐ \$182

(**) long run hist



$P(\text{coming out ahead } Ⓐ) = P(S > \$0) = ?$

$$\mu = \frac{(\$-1) + (\$-1) + \dots + (\$-1) + (\$+35)}{38}$$

$$= \frac{37(-\$1) + 2(+\$35)}{38} = \frac{-\$2}{38}$$

$$\doteq -\$0.0526$$

with every \$1 bet on single #,

I expect to gain $\mu = \boxed{-\$0.05}$, utility

give or take $\sigma = \$5.76$

$$\sigma = \sqrt{\frac{[(\$-1) - (-\$0.05)]^2 + \dots + [(\$-1) - (-0.05)]^2 + [(\$+35) - (-0.05)]^2}{38}}$$

math fact | if pop. has only 2 values in it

$$\sigma = \underset{(\sigma)}{\text{pop. SD}} = \left[\begin{pmatrix} \text{larger} \\ \text{value} \end{pmatrix} - \begin{pmatrix} \text{smaller} \\ \text{value} \end{pmatrix} \right] \sqrt{\begin{pmatrix} \text{prop. of} \\ \text{larger} \\ \text{values} \end{pmatrix} \begin{pmatrix} \text{prop. of} \\ \text{smaller} \\ \text{values} \end{pmatrix}}$$

here $\sigma = \underbrace{\left[(+\$35) - (-\$1) \right]}_{+\$36} \sqrt{\frac{1}{38} \cdot \frac{37}{38}}$

$= \$5.76$

(your net gain after 1000 \$1 bets on single #) is $\boxed{\text{exactly like}}$ (the sum S of n = 1,000 IID draws from this population) *

real world

essence of applied math

simulation (1950) → math & computing (probability model) ← (4000 years) range of possible values for $\$S$

$$-\$1{,}000 \qquad (\,) \leftarrow \$-53 \pm 2(\$182) \qquad +\$35{,}000$$

$(+35)\frac{1}{38}$ $\qquad \frac{1000}{38} \doteq 26$ $\qquad 26(\$+35) + 974(\$-1)$

$(\$-1)\left(\frac{37}{38}\right)$

$= \$-64$

$28(\$+35) + 972(\$-1)$

$= \$+8$

$\begin{pmatrix}\text{long}\\ \text{run}\\ \text{mean}\\ \text{of } \$\end{pmatrix} = \begin{pmatrix}\text{expected}\\ \text{value}\\ \text{of } \$\end{pmatrix} = \text{EV of } \$$ $\quad ((12.45))$

$$= E_{IID}(\$) = n \cdot \mu = \begin{pmatrix}\#\\ \text{rows}\end{pmatrix} \cdot \begin{pmatrix}\text{p.y.}\\ \text{mean}\end{pmatrix}$$

$$= (1000)(\$-0.0526) = \$-52.60$$

$$\begin{pmatrix} \text{long run SD of } \$ \end{pmatrix} = \begin{pmatrix} \text{standard error of } \$ \end{pmatrix} = \underline{\underline{SE \text{ of } \$}}$$

$$= \boxed{SE_{IID}(\underline{\underline{\$}}) = \frac{\sigma \sqrt{n}}{1} = \sigma\sqrt{n}}$$

give or take for $\$$ = noise level in predicting $\$$

N	X
μ	X
σ	as $\sigma \uparrow$ $SE(\$) \uparrow$
n	as $n \uparrow$ $SE(\$) \uparrow$

formulas (2)

after 1000 \$1 bets in ~~single~~ single #, I expect to have gained (-\$53), around $E(\$)$, give or take _____ ← $SE(\$)$

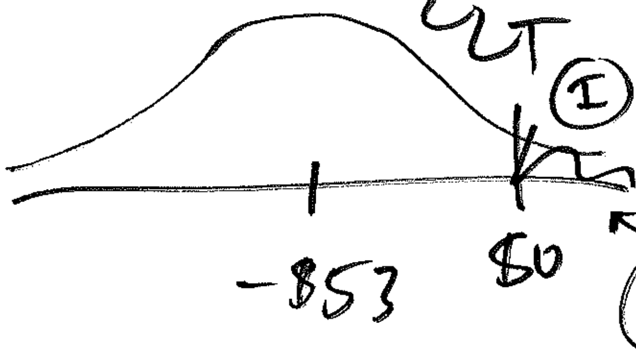
$$SE_{IID}(\$) = \sigma\sqrt{n} = (\$5.76)\sqrt{\#\text{draws}}$$

$$= (\$5.76)\sqrt{1000} \doteq \$182$$

**

SE $182

CLT I



long run hist of $

(1710) (de Moivre)

Central Limit Theorem

+0.29

L - 124

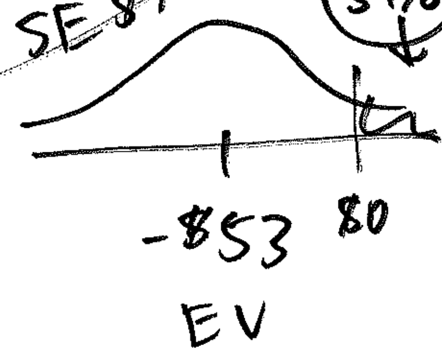
$$\frac{\$0 - (-\$53)}{\$182} = +0.29$$

Split B

SE $127

34%

long run hist. of $



$$\frac{\$0 - (-\$53)}{\$127} = +0.42$$

P(coming out ahead)